Bayesian Compressed Sensing Based Dynamic Joint Spectrum Sensing and Primary User Localization for Dynamic Spectrum Access

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***Abstract*—In cognitive radio and dynamic spectrum access network research, most of current work on spectrum sensing focus on the detection of existence of the spectrum holes for secondary user to harness. However, in a more sophisticated CR, the secondary user needs to detect more than just the existence of primary users and spectrum holes: e.g., the transmission power and location of the primary users. In our previous work, we combined the spectrum sensing and primary user power/localization detection together, and developed a joint primary user detection and power/localization detection algorithm via compressed sensing (CS). By employing compressed sensing, the data sampling rate of the spectrum sensors is significantly reduced. However, if the sampling rate is too low, the compressed sensing algorithm will not provide accurate spectrum sensing and power/localization estimates. Since the sparsity in the frequency domain is dynamically changing, it is unfeasible to set a predetermined sampling rate. In this paper, we extend our previous work to employ the Bayesian Compressed Sensing (BCS) to improve the reconstruction results and dynamically determine the sampling requirement. Specifically, the BCS algorithm provides an "error bar" along with the reconstruction of the target vector. This "error bar" can then be used to determine if the current sampling rate is sufficient. When the spectrum environment changes, the "error bar" will change accordingly, giving us direction to increase or decrease the sampling rate. Simulation results including the sampling ratio, the miss detection probability (MDP), false alarm probability (FAP) and reconstruction probability (RP) confirm the effectiveness and robustness of the proposed method.**

I. Introduction

In Cognitive Radio [1][2] and Dynamic Spectrum Access Network [3], spectrum sensing [4][5] is an important process for the secondary user (SU) to detect spectrum usage status to avoid interference to the primary user (PU). Compressed Sensing (CS) [6][7] technology is adopted as an important method for spectrum sensing since wireless signals in open spectrum networks are typically sparse in frequency domain [8], and the CS algorithm is a powerful technique for acquiring and reconstructing a signal utilizing the prior knowledge that it is sparse or compressible with limited (incomplete) measurements. For overlay CR technology, once there is a spectrum hole called *white* spectrum (unused spectrum region), secondary user can utilize it to transmit, therefore the task for spectrum sensing is to detect if there is primary user or not. In [9] and [10], authors proposed to treat the spectrum sensing for overlay CR as a binary event detection and apply compressed sensing technology to detect the existence of primary user as binary event.

On the other hand, for a more sophisticated cognitive radio such as hybrid overlay/underlay cognitive radio [11][12], secondary user can also use the *gray* spectrum (occupied by primary user), and it is required to limit the power of secondary user to avoid harmful interference to primary user. Specifically, after the transmission, the power of the secondary user at primary user's receiver should be below the primary user's interference tolerance level. Hence, besides the existence of the primary user, it is also necessary to find more information related to the primary user, such as the power of the primary user, the distance between the secondary user and primary user (or location of primary user). As a directly result, the spectrum sensing task is not limited to a binary decision. Instead, power and location of the primary user should also be detected.

In our previous work [13], we combined the primary user detection problem and the power/localization detection of primary user problem together and formulated it as a sparse matrix problem. We proposed to use compressed sensing technology to reconstruct the primary users' information in this sparse vector, using the assumption that the reconstructed information vector is sparse. In [13], we successfully detected the existence of primary users, their transmission powers and locations with significantly reduced sampling rate required at the spectrum sensors, thanks to the compressed sensing algorithm and the sparsity of the spectrum. However, how to set the sampling rate in [13] was unknown. If the sampling rate is set too low, the estimation accuracy will suffer tremendously. If the sampling rate is set unnecessarily high, more burden is put on the spectrum sensors. We want as low sampling rate as possible but not too low. Additionally, since the sparsity of the spectrum environment is constantly changing, it is unfeasible to predetermine the sampling rate.

In this paper, we extend our previous work in [13] to dynamically determine the necessary sampling rate of the compressed sensing algorithm. Specifically, we employ the Bayesian Compressed Sensing (BCS) [14] algorithm in our spectrum sensing and power/localization estimation algorithm. The BCS algorithm provides a unique measurement of the accuracy of the reconstructed vector called “error bars”. With the “error bars”, it is now feasible to determine if the sampling rates are sufficient, and dynamically modify (increase/reduce) the sampling rates according to the environment change.

Simulation results including the miss detection probability (MDP), false alarm probability (FAP) and reconstruction probability (RP) confirm the effectiveness and robustness of the proposed method compared with the previous work.

The rest of the paper is organized as follows: In section II, we introduce the system model of the joint spectrum sensing and localization problem. The joint spectrum sensing and primary user localization problem in previous work is briefly reviewed in Section III. Section IV describes the proposed Bayesian compressed sensing applying in the joint spectrum sensing and localization problem. Section V demonstrates the simulation results which confirm the effectiveness and robustness of the proposed method, and conclusion is given in Section VI.

II. System Model

To illustrate the joint spectrum sensing and the primary user localization, the system model includes a number of PUs in one geographical area, a group of available frequency domain “channels”, and a group of spectrum sensors that sense the spectrum environment. Suppose there are N_{ch} channels available for a wideband wireless network in one area. There exist N_p primary users occupying N_p different channels with power spectrum density (PSD), and these PUs are distributed over a large geographic area. For more general cases, different PUs at different locations can occupy the same band, which means the spectrum from one single band can be contributed from differently located PUs. The parameters of the system model change dynamically, including the number of PUs, the locations and channel occupation.

Now we distribute N_c RF sensors in this area to detect all these channels. The measurements from all the sensors are sent to a fusion center. For the transmission of secondary users, it is needed to know which channel is occupied, and which is not. More important, for the occupied channel, it is also desired to detect the power and location of the primary user. Hence, our task is to detect which channels are occupied, and also identify the primary users’ transmission powers and locations. Such a problem is the combination of traditional spectrum sensing problem and localization problem.

Path loss model is applied to simulate the received power related to the distance, and the power will be reduced according to the channel frequency and distance between the PUs and sensors, and the relationship between them can be expressed as:

$$L(f,d) = P_0 + 2\log_{10}(f) + 10n\log_{10}(d)(dB) \tag{1}$$

where $L(f,d)$ is the path loss in dB scale and P_0 is a constant associated to the antenna gain; f is the frequency corresponding to the channel (here we assume f is the center frequency of the channel, and for N_{ch} channels there are $\{f_0, f_1, ..., f_{N_{ch}-1}\}$) and n is the path loss exponent; d presents the distance between the transmission node and receiving node. Here, the parameters to compute the path loss (channel frequency and location of PUs) are unknown and to be determined as well. Therefore, the task is to use the received signal after transmission through unknown channel to reconstruct the channel occupation, location and the transmitted signal power.

III. Joint Compressed Sensing based Spectrum Sensing and Localization

In our previous work [13], we have demonstrated that the information reconstruction can be expressed as a matrix problem. Here we briefly review the reconstruction process.

First of all, the time domain measurement $\vec{d}$ to frequency domain measurement $\vec{D}$ can be expressed using linear *Inverse Discrete Fourier Transform (IDFT)*

$$\vec{d} = \mathbf{F}^H \vec{D} \tag{2}$$

where $\mathbf{F}^H$ is the normalized *IDFT* matrix.

Next, we discretize the model by assuming the primary users only locate at discrete grid points (x_m, y_n) and $x_m \in \{0, \Delta x, 2\Delta x, ..., (M-1)\Delta x\}, y_n \in \{0, \Delta y, 2\Delta y, ..., (N-1)\Delta y\}$, where Δx and Δy are the resolutions in x-axis and y-axis. The N_c CR nodes are located at $(a_1, b_1), (a_2, b_2), ..., (a_{N_C}, b_{N_C})$, which are not needed to be discrete. Combining all the information of PU, it is easy to show the linear relationship between the received signal and the information of PUs (details are shown in [13]):

$$\begin{aligned}
&\vec{Y} = \mathbf{L} \cdot \vec{P} \qquad (3)\\
&\vec{Y} = [\vec{Y}_{0,0}, \vec{Y}_{0,1}, \vec{Y}_{0,N_{ch}-1}\\
&\quad \vec{Y}_{1,0}, \vec{Y}_{1,1}, \vec{Y}_{1,N_{ch}-1}, ...\\
&\quad \vec{Y}_{N_c-1,0}, \vec{Y}_{N_c-1,1}, ..., \vec{Y}_{N_c-1,N_{ch}}]^T\\
&\mathbf{L} = \begin{bmatrix} \vec{L}_0 \\ \vec{L}_1 \\ \vdots \\ \vec{L}_{N_C-1} \end{bmatrix}\\
&\vec{P} = [P_{0,0}, P_{0,2}, ..., P_{0,MN-1},\\
&\quad P_{1,0}, P_{1,1}, ..., P_{1,MN-1}, ...,\\
&\quad P_{N_{ch}-1,0}, P_{N_{ch}-1,1}, ..., P_{N_{ch}-1,MN-1}]^T\\
&\vec{L}_k = diag\{\vec{l}_{k,0}, \vec{l}_{k,1}, ..., \vec{l}_{k,N_{ch}-1}\}\\
&\vec{l}_i = [l_{k,i,0}, l_{k,i,1}, ..., l_{k,i,MN-1}];
\end{aligned}$$

where $P_{i,m+n\times M}$ presents the power from the PU at location (x_m,y_n) transmitting at channel i, and $m \in \{0, 1, ..., M-1\}$, $n \in \{0, 1, ..., N-1\}$, $i \in \{0, 1, ...N_c - 1\}$, and $\vec{P}$ includes all the information needed to be reconstructed. Since the information vector $\vec{P}$ contains all the combination of the location and the channel occupation, the scenarios include the case that differently located PUs can occupy the same channel. $l_{k,i,m+n\times M}$ is the fading gain from the PU at location (x_m, y_n)

occupying at channel i to the k^{th} sensor node

$$l_{k,i,m+n\times M} = 10^{L(f_i,d(m,n,k))/10} \tag{4}$$

where $L(f_i, d(m,n,k))$ is defined in Eq. (1) and $d(m,n,k) = \sqrt{(x_m - a_k)^2 + (y_n - b_k)^2}$ presents the distance between the PU and the sensor node.

Finally, combing the two matrix expressions mentioned above, a one-step matrix problem is built. The received signal sample vector in time domain for all the N_C CR nodes is

$$\vec{d} = \mathbf{F}_D^H \mathbf{L}\vec{P} = \mathbf{\Phi}\vec{P} \tag{5}$$

where $\mathbf{F}_D^H$ is the diagonal combination of original normalized *IDFT* matrix:

$$\begin{aligned}\mathbf{F}_D^H = [&\mathbf{F}^H, \mathbf{0}, \mathbf{0}, ..., \mathbf{0};\\ &\mathbf{0}, \mathbf{F}^H, \mathbf{0}, ..., \mathbf{0};\\ &...;\\ &\mathbf{0}, \mathbf{0}, \mathbf{0}, ..., \mathbf{F}^H];\end{aligned} \tag{6}$$

where $\mathbf{0}$ presents the zero matrix which has the same size as $\mathbf{F}$.

IV. Compressed Sensing Background

A. Compressed Sensing

Compressed Sensing is a technique for acquiring and reconstructing a signal utilizing the prior knowledge that it is sparse or compressible with limited (incomplete) measurements. Specifically, if $\vec{P}$ is the $N \times 1$ sparse signal vector with K non-zero elements, the $\vec{d}$ presents $M \times 1$ measurement vector which has relationship of $\vec{P}$ as:

$$\vec{d} = \mathbf{\Phi}\vec{P} \tag{7}$$

where $\mathbf{\Phi}$ is the measurement matrix which defines the relationship between $\vec{P}$ and $\vec{d}$.

When $\mathbf{\Phi}$ satisfies Restricted Isometry Property (RIP), it only needs $M = O(K \log(N/K))$ measurements to recover $\vec{x}$ with very high probability. The recovery algorithm can be expressed as:

$$\begin{aligned} min||\vec{P}||_1 = min\sum_i |P_i| \\ s.t. \quad \vec{d} = \mathbf{\Phi}\vec{P}\end{aligned} \tag{8}$$

which is called *Basic Pursuit*. On the other hand, when there exists noise in the measurement, *Second-Order Cone Program* can be used as the optimization strategy with the threshold ϵ:

$$\begin{aligned} min||\vec{P}||_1 = min\sum_i |P_i| \\ s.t. \quad ||\vec{d} - \mathbf{\Phi}\vec{P}||_2 \le \varepsilon\end{aligned} \tag{9}$$

Besides the theoretical optimization methods, signal reconstruction from the set of projections is obtained from the solution of a simplex convex optimization problem that can be solved using some fast iterative algorithms [15][16][17][18].

B. Bayesian Compressed Sensing

Bayesian Compressed Sensing [14] is a Bayesian framework for solving the inverse problem of compressed sensing. The basic BCS algorithm applied the relevance vector machine (RVM) [19], and it is also extended by marginalizing the noise variance with improved robustness. Besides providing a better solution based on Bayes rule, the most important issue we consider for this paper is: a stopping criterion for determining when a sufficient number of CS measurements have been performed.

Specifically, the BCS views the compressed sensing process as a linear regression, and assigns a *prior* distribution $p(\vec{P}|\vec{\beta}) = \prod_{i=1}^{N} \mathbb{N}(P_i|0, \beta_i^{-1})$ to the unknown vector $\vec{P}$, where $\mathbb{N}(P_i|0, \beta_i^{-1})$ presents the Gaussian distribution with zero mean and variance β_i^{-1}; meanwhile the measurement vector $\vec{d}$ is also assigned the distribution $p(\vec{d}|\vec{P}, \beta_0)$. Since $\vec{d} = \mathbf{\Phi}\vec{P}+\vec{n}$, it is clear that $p(\vec{d}|\vec{P}, \beta_0)$ is Gaussian distribution with variance β_0^{-1} as well. $\vec{\beta} = [\beta_1, \beta_2, ..., \beta_N]$ and β_0 are the hyperparameters associated with the weight of each estimate which dictate the prior.

Using Gaussian likelihood and Bayes' rule, the conditional probability density function for $\vec{P}$ can be represented as:

$$p(\vec{P}|\vec{d}, \vec{\beta}, \beta_0) = (2\pi)^{-\frac{N}{2}} |\mathbf{\Sigma}|^{-\frac{1}{2}} \exp(-\frac{1}{2}(\vec{P}-\vec{\mu})^T \mathbf{\Sigma}^{-1}(\vec{P}-\vec{\mu})) \tag{10}$$

where

$$\begin{aligned}\vec{\mu} &= \beta_0 \mathbf{\Sigma}\mathbf{\Phi}^T\vec{d}\\ \mathbf{\Sigma} &= (\mathbf{B} + \beta_0\mathbf{\Phi}^T\mathbf{\Phi})^{-1}\\ \mathbf{B} &= diag\{\vec{\beta}\}\end{aligned} \tag{11}$$

and it is clear this conditional probability density function is a Gaussian distribution $\mathbb{N}(\vec{\mu}, \mathbf{\Sigma})$.

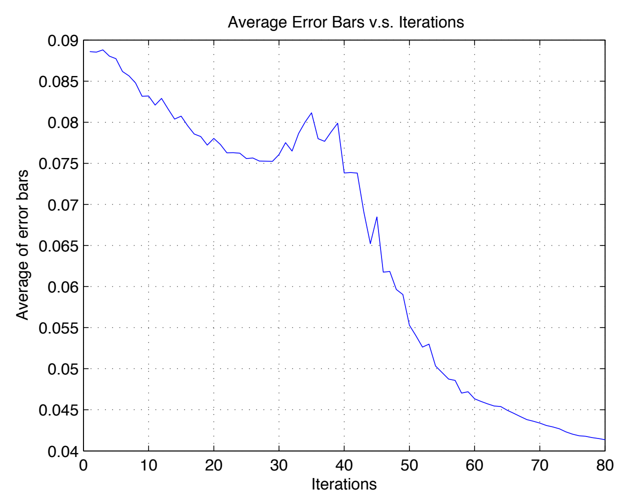


Fig. 1. Average "error bars" v.s. the number of measurements

It is important to note that $\mathbf{\Sigma}$ represents the variance of the reconstructed parse signal vector $\vec{P}$, which indicates the "error bars" of the reconstruction. From this "error bars", it is easy to determine if the number of measurements is enough for the requirement. Fig. 1 shows the average of "error bars" (reconstruction variance) versus the number of measurements. It is evident that when the number of measurements increases, the average variance decreases. When the average variance reaches the requirement, the number of measurements is enough (e.g., 70 measurements are enough to obtain the average of reconstruction variance as low as 0.045 in Fig. 1).

The iterative increasing of measurement process can help to minimize the measurements, and not exceed the maximum sampling ratio we can accept. If the maximum sampling ratio is not given, the BCS method will collect as much data as needed to meet the requirement for the average "error bar". Here is the iterative measurement process:

1) Setting the initial number of measurements
2) Run the BCS to reconstruct the information vector
3) Check if the "error bars" meets the requirement E_B or the current measurement reaches the maximum sampling ratio.
   - If it is, then stop
   - If it is not, then add l measurement, and go to step 2).

The parameter l can be chosen according to the requirement of the system.

C. Real Compressed Sensing

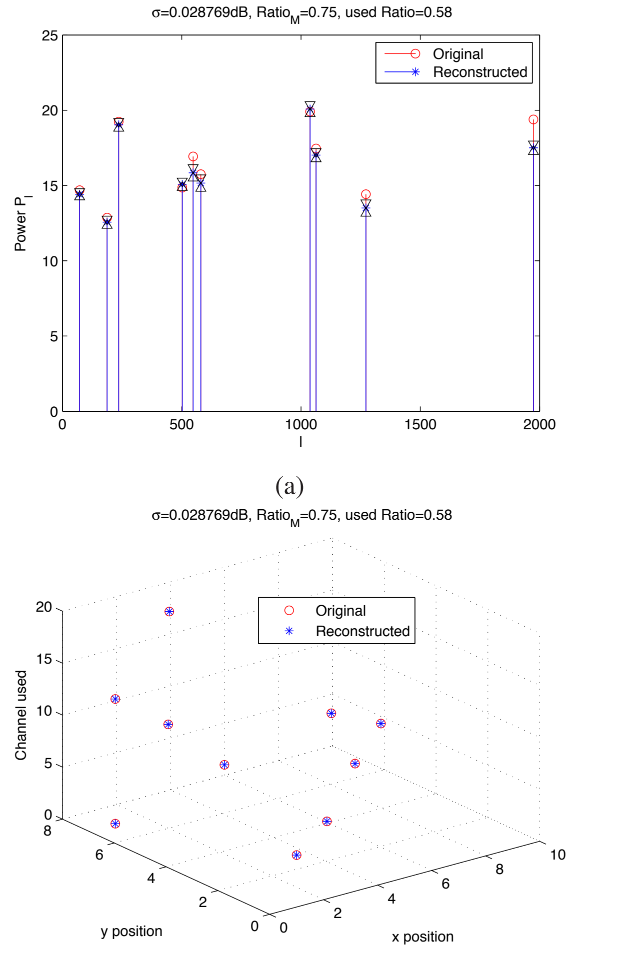
Utilizing real Compressed Sensing to recover $\vec{P}$, it is easy to find the real values expression of Eq. (5):

$$\begin{bmatrix} real(\vec{d}) \\ imag(\vec{d}) \end{bmatrix} = \begin{bmatrix} real(\mathbf{\Phi}) & -imag(\mathbf{\Phi}) \\ imag(\mathbf{\Phi}) & real(\mathbf{\Phi}) \end{bmatrix} \begin{bmatrix} real(\vec{P}) \\ imag(\vec{P}) \end{bmatrix} \tag{12}$$

Hence, by applying CS technology with the observations in time domain, the channel occupation, the power and location of these PUs can be reconstructed. To better understand the proposed joint reconstruction, we can also treat it as a 3-D image recovery problem. The x-axis and the y-axis of this unknown image present the location of the primary users, and the z-axis denotes the channel occupation status. The value of this image at each 3-D position corresponds to the transmission power. The purpose of the proposed method is to recover this 3-D image by some sparse observations.

V. Simulation Results and Analysis

In this section, we examine the effectiveness of the proposed method to recover the information of the primary users in one geographical area. In all the simulations, total of 20 channels in $50 + [1, 2, ..., 20]$ MHz can be occupied by primary users, and these primary users are located in a $(M \times N)$ area with $M = 10, N = 10$ and resolution $\Delta x = 0.1, \Delta y = 0.1$. The CR nodes are also located in this range. The power of each primary user is set to be a random value in range $[10, 20]$. Denote K to be the number of observations from all the CR nodes in time domain, and the average sampling ratio can be defined as $ratio = K/(2 \times N_c \times N_{ch})$, which reflects the reduced number of samples used with reference to the



(b)

Fig. 2. Construction result 1: (a) Information Reconstruction of Primary Users (b) 3-D Reconstruction Figure

number of samples completely needed, and the number 2 is here because the complex values consists of real number and imaginary number as shown in Eq. (12). It is important to note that the $ratio$ is associated with the number of sensor nodes. When the number of sensor nodes increases, more measurement can be obtained using the same ratio. In the simulation, we set 5 CR nodes to do the localization task.

To demonstrate the performance of the information reconstruction, three features are defined.

1) The miss detection probability (MDP):
$$MDP = \frac{N_p - N_{detected}}{N_p} \tag{13}$$
where $N_{detected}$ presents the number of correct detections (including the channel occupation and location).
2) The false alarm probability (FAP):
$$FAP = \frac{N_d - N_{detected}}{N_d} \tag{14}$$
where N_d denotes the number of detections.
3) The reconstruction probability (RP): It presents the

probability of accurate reconstruction. If the power reconstruction is within the range of $\pm 10\%$ to the original power, we consider the reconstruction to be accurate, vice versa.

The MDP and FAP illustrate the spectrum sensing and localization performance, while the RP presents the power reconstruction performance.

In practice, a threshold P_T can be set to eliminate some detections. When the reconstructed power $\hat{P}$ is lower than P_T, we make decision that there is no primary user. The effect of P_T has been illustrated in [13]: When the threshold P_T is lower, the miss detection probability will be reduced since the system becomes more sensitive and even a very small reconstruction power can be determined as a sign of existence of PU. However, a lot of false detections will be generated which will increase the FAP. On the other hand, when P_T becomes higher, it may eliminate more false detections. In the rest of simulations, we set $P_T = 2$.

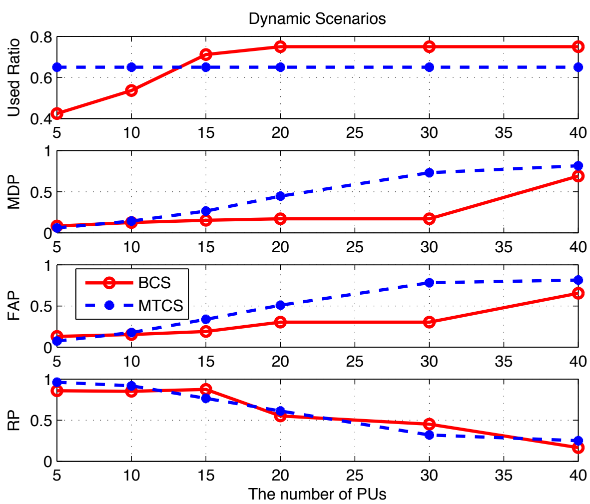


Fig. 3. Performance v.s. the number of PUs in Dynamic Scenario

Fig. 2 presents the reconstruction when 5 CR nodes are located at $(13.4160, 8.5087)$, $(10.5119, 9.8209)$, $(19.4252, 9.5925)$, $(2.6901, 4.6243)$, $(1.6220, 3.9095)$ with maximum $ratio_M = 75\%$ and measurement noise $\sigma = 0.0288$, and the P_T is set to be 2. Besides the $ratio_M$, the other condition to stop the iteration is when the average "error bar" reaches as lower as $E_B = 0.15$. Fig. 2(a) shows the original information vector $\vec{P}$ which contains the power, location, channel occupation of primary users marked with red circle, compared with the reconstructed $\hat{\vec{P}}$ in blue star. The black upper triangle and lower triangle present the "error bars", which indicate the variance of this reconstruction. Fig. 2(b) presents the 3-D figure comparison of the original and recovered information according to the location and channel occupation, which can be easily obtained from the vector $\vec{P}$ and $\hat{\vec{P}}$. It is clear that the reconstructed vector (blue star) matches the original vector (red circle) well, and the actual used ratio $ratio_A = 58\%$ is less than the given maximum ratio 0.75%. The MDP and FAP are equal to zero, indicating perfect detection.

To illustrate the dynamical environment, the number of PUs changes with the same setting: maximum ratio $ratio_M = 75\%$. Fig. 3 illustrates the actual sampling ratio, MDP, FAP, and RP versus the number of PUs. BCS method and the MTCS (multi-task CS) method with a fixed sampling ratio 65% are compared in this figure. When the average "error bar" reaches as lower as $E_B = 0.15$, the iteration process of BCS will stop. The RP performances of both system are almost the same, illustrating the benefit of the BCS method.

For small number of PUs, the actual sampling ratio applied for the BCS method is lower that the maximum ratio 75% to meet the requirement (e.g., about 52% sampling ratio is applied for BCS method when there are 10 PUs), which requires much less measurements compared to the MTCS method. Meanwhile, the performance of the BCS is similar to that of the MTCS. Hence, it is evident that when small number of PUs are occupying the spectrum, BCS uses much less measurements than the MTCS method with the same performance requirement.

When the number of PUs increases, the sampling ratio applied in BCS also increases, and when the actual sampling ratio is as large as the maximum ratio, no more measurements are obtained. The sampling ratio is automatically changing in BCS according to the environment with a fixed E_B.

When the number of PUs becomes large (e.g., in the range $[15, 30]$), the performance of MTCS method with the fixed sampling ratio degrades since the sampling ratio 65% is not enough; while the BCS dynamically adapts to appropriate sampling ratio and maintains the good FAP and MDP performances.

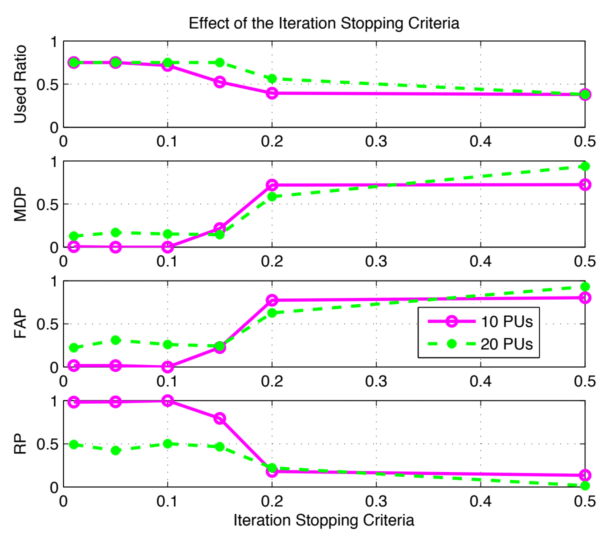


Fig. 4. Performance v.s. iteration stopping criteria in Dynamic Scenario

In Fig. 4, the effect of the iteration stopping criteria E_B is presented in the cases of 10 PUs (purple line marked with circle) and 20 PUs (green dot line marked with star) for stopping criteria $E_B \in \{0.01, 0.05, 0.10, 0.15, 0.2, 0.5\}$.

For both curves, we draw the same conclusion: When E_B is too small, it is not easy to reach that small average "error bar", hence more measurements are needed. For example, when stopping criteria is $E_B = 0.01$, the actual sampling ratio reaches the maximum ratio 75%. The performance for this scenario is very good. When we don't require very good performance, the E_B can be set larger and the actual sampling ratio will be much lower than the maximum ratio. However, the performance will also degrade due to the lack of measurements while the performance has already met the requirement E_B.

With the constraints from E_B and the maximum sampling ratio, the BCS method can minimize the number of measurements and not exceed the predefined maximum sampling ratio. In practice, if the system focuses on better performance with minimal measurements, only E_B is needed to determine when to stop the iterations, otherwise the requirement of maximum sampling ratio can be added.

VI. Conclusion

In this paper, based on our previous work of jointly detecting existence of primary users and the power/location of them via compressed sensing, we employ the Bayesian Compressed Sensing to improve the reconstruction results and dynamically determine the sampling requirement. Specifically, the BCS algorithm provides an "error bar" along with the reconstruction of the target vector. This "error bar" can then be used to determine if the current sampling rate is sufficient. When the spectrum environment changes, the error bar will change accordingly, giving us direction to increase or decrease the sampling rate. Two iteration stopping criteria including the average "error bar" and the maximum sampling ratio are applied in the system to dynamically change the number of measurements for better performance and less measurements. Simulation results including the actual sampling ratio, miss detection probability, false alarm probability and reconstruction probability confirm the effectiveness and robustness of the proposed method.

Acknowledgment

This work is supported by National Science Foundation under Grants No. 0708469, No. 0737297, No. 0837677, the Wright Center for Sensor System Engineering, and the Air Force Research Laboratory. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the funding agencies.

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