Multi-Resolution Bayesian Compressive Sensing for Cognitive Radio Primary User Detection

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***Abstract*—Current Cognitive Radios are limited in their operational bandwidth by existing hardware devices, much of the extensive theoretical work on spectrum sensing is impossible to realize in practice over a wide frequency band. To alleviate the sampling bottleneck, some researchers have begun to use a technique called Compressive Sensing (CS), which allows for the acquisition of sparse signals at sub-Nyquist rates, in conjunction with CRs. These researchers have sequentially combined the two techniques: first acquiring compressed samples, then reconstructing the Nyquist rate signal, and lastly performing CR spectrum sensing on the reconstructed signal. While CS alleviates the bandwidth constraints imposed by front-end ADCs, the resulting increase in computation/complexity is non-trivial, especially in a power-constrained mobile CR. In addition, the computation time of the signal reconstruction introduces significant delay into the spectrum sensing operation. This motivates us to look at different ways to reduce computational complexity while achieving the same goals.**

In this paper, we will demonstrate how utilizing a Bayesian Compressive Sensing (BCS) framework can achieve the sampling reduction advantage of Compressive Sensing with significantly less computational complexity. Our key observation is that the CR does not have to reconstruct the entire signal because it is only interested in detecting the presence of Primary Users. Our BCS PU detection algorithm takes advantage of this observation by estimating signal parameters directly from the compressed signal, thereby eliminating the reconstruction stage and reducing the computational complexity. In addition, the BCS framework provides a measure of the quality of estimation allowing the system to optimize its data acquisition process to always acquire the minimum number of compressed measurements, even in a dynamic spectral environment.

I. Introduction

THE motivation and techniques for spetrum agile Cognitive Radios (CR)s to efficiently utilize unoccupied licensed spectrum have been well documented in literature, but due to hardware and sampling constraints, CRs in practice are often limited to a restricted frequency range, severely limiting their usefulness. As a result, most of the CR results derived in literature [1][2][3] are impossible to realize in practice over a wide frequency band.

Recently, an area that has demonstrated a the potential to alleviate the sampling bottleneck in wideband communications is Compressive Sensing (CS), which asserts that the one can recover sparse signals at sub-Nyquist rates [4][5]. CS relies on this principle of sparsity, so that a concise representation of the signal is possible when expressed in the proper basis. Due to the low percentage of spectrum occupancy by Primary Users, which originally motivated the development of CRs, wireless communication signals in open-spectrum networks are typically sparse in the frequency domain, allowing us to use compressive sensing to alleviate the sampling bottleneck.

While CS is a powerful technique, it does not allow the CR to sample at low rates for free; the resulting increase in computation/complexity is non-trivial, especially in a power-constrained mobile Cognitive Radio. This motivates our development of a scheme which can perform the functions of a CR (detect primary users in a wide frequency range) with as few samples as possible while incurring minimum computation/complexity. Previous works [6][7] have also realized the synergies between Compressive Sensing and Cognitive Radios and attempted to capitalize by combining both processes in a sequential fashion. In both works, the sensing bottleneck was alleviated with compressive sensing, which was used to capture the signal at a reduced rate. The entire signal was reconstructed from the compressed samples to the Nyquist number of samples, from which the spectrum sensing was then performed. Both papers failed to take into account the significant computational complexity necessitated by the reconstruction technique and the additional delay which this computation incurred, issues which we will directly address and solve in this paper. In addition, the algorithms in [6][7] have to have *a priori* knowledge of the sparsity of the signal in order to determine how many measurements (M) are required to ensure perfect reconstruction. Without this knowledge, they had no metric to assess the quality of their reconstruction. In practice, this knowledge is generally unavailable and the sparsity of the signal will in fact change significantly over time.

In this paper, we build upon the key observation that we made in our earlier work [8] : The fundamental task of a CR is not to analyze the entire signal, rather it is to estimate the presence of Primary Users, thus if we can directly estimate the Primary User signals from the compressed measurements, the reconstruction stage can be completely eliminated. In [8] we showed that this objective could be accomplished using a Bayesian formulation to reconstruct the signal. The main contribution in [8] was to utilize Bayesian Compressive Sensing (BCS) as the CR Spectrum Detector, estimating the parameters of the primary users directly from the compressed signal.

While powerful, the BCS framework requires a point-by-point modelling of the entire signal. To further alleviate this complexity in this paper, we will introduce a multi-resolution wavelet hiearchical scheme in which we model a low-resolution wavelet-decomposed signal rather than the original resolution signal. This Multi-Resolution Bayesian Compressive Sensing (MRBCS) framework lends itself well to an iterative optimization strategy to A) acquire the minimum number of samples in a dynamic sparsity model without any *a priori* assumptions, B) only acquire detailed information on the sparse energy bands. This overall strategy succeeds in meeting our primary goal: reducing the computational complexity in Cognitive Radios utilizing Compressive Sensing.

The remainder of the paper is organized as follows. In Sec. II, we'll begin by defining a signal model which demonstrates sparsity so that compressive sensing can be applied. Sec. III briefly reviews our work in [8] to demonstrate how a Bayesian signal inversion framework essentially functions as a wideband CR, performing signal reconstruction and spectrum sensing in one step, reducing complexity. Sec. IV will discuss how the complexity of this system can be further alleviated with a multi-resolution wavelet hierarchical scheme. Finally, simulation results for our MRBCS method, along with comparisons to other algorithms, including our algorithm in [8], will be presented in Sec. V with an emphasis on computation complexity, speed, and accuracy. Sec. VI presents our final thoughts and concludes the paper.

II. Signal Model

We'll begin by defining the signal model, which demonstrates sparsity so that compressive sensing can be applied [10]. Suppose that a Cognitive Radio wishes to analyze a signal $x(t)$ that spans a total of B_{tot} Hz in the frequency range $[f_0, f_N]$. The range is occupied by $j \in [0, K]$ users, each occupying arbitrary disjoint bands of equal bandwidth B. In order to sample below the Nyquist rate, we assume that $2jB < f_{nyq}$ (if this condition is not satisfied, CS cannot be used), so the signal is sparse in the frequency domain. Suppose that the time window for sensing is $t \in [0, NT_{nyq}]$, where T_{nyq} is the Nyquist sampling period and assuming Nyquist theory, N samples are needed to recover $x(t)$ perfectly without aliasing. A digital receiver may not be able to sample at this rate for a very wideband and so we apply Compressive Sensing to obtain low-rate samples in the following form.

$$\mathbf{y}_t = \mathbf{S}\mathbf{x}_t = \mathbf{S}\mathbf{F}_N^{-1}\mathbf{X}_f = \mathbf{A}\mathbf{X}_f \quad (1)$$

where $\mathbf{S}$ is an MxN projection matrix and $\mathbf{x}_t$ represent the Nyquist rate Nx1 vector with $x_t[n] = x(t)|_{t=nT_0}, n = 1, ..., N$. The columns of $\mathbf{S}$ are a set of matched filters or set of basis signals and the compressed measurements $y_t[m]|_{m=1}^{M}$ are essentially the projection of $\mathbf{x}_t$ onto the basis. The relationship between the measurement lengths is $j << M < N$. $\mathbf{F}_N$ denotes an N point unitary DFT matrix mapping the time domain signal to the frequency domain where $\mathbf{X}_f$ is sparse. $\mathbf{A}$ = $\mathbf{S}\mathbf{F}_N^{-1}$, which is MxN, allows us to express the relationship between the compressed time domain signal, $\mathbf{y}_t$, to the Nyquist rate frequency domain sampled signal $\mathbf{X}_f$.

III. Bayesian Modeling of CS Inversion

We model the unknown signal, $\mathbf{X}_f$, as a random process and assign it a *prior* distribution $p(\mathbf{X}_f|\boldsymbol{\beta})$. The observation of the compressed samples is also a random process with conditional distribution $p(\boldsymbol{y}_t|\boldsymbol{X}_f, \beta_0)$. The measurements $\boldsymbol{y}_t$ may also be noisy and so we account for measurement noise by adding $\boldsymbol{n}_f$, to our model, giving us $\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{X}_f + \boldsymbol{n}_f$ where $\boldsymbol{n}_f$ is zero mean uncorrelated Gaussian noise with variance σ^2. $\boldsymbol{\beta}$ and β_0 are hyperparameters associated with each estimate's weight; β is the inverse variance (precision) of a Gaussian pdf and $\beta_0 = 1/\sigma^2$ is the noise precision. The conditional distribution of the observation thus becomes a Gaussian liklihood model

$$p(\boldsymbol{y}_t|\boldsymbol{X}_f, \beta_0) = (2\pi/\beta_0)^{-M/2} exp(-\frac{\beta_0}{2}||\boldsymbol{y}_t - \boldsymbol{A}\boldsymbol{X}_f||^2) \quad (2)$$

which we seek to estimate. Thus the CS problem of reconstructing the sparse frequency domain samples $\boldsymbol{X}_f$ becomes a linear regression problem with the constraint that $\boldsymbol{X}_f$ is sparse. Assuming that we know the random projection matrix $\boldsymbol{A}$, the goal is to estimate the parameters of $\boldsymbol{X}_f$ and σ^2. We begin by looking at point modeling, which is the traditional BCS method of estimating parameters [9]. We first define a zero-mean Gaussian prior on each element of $\boldsymbol{X}_f$.

$$p(\boldsymbol{X}_f|\boldsymbol{\beta}) = (2\pi)^{-N/2} \prod_{n=1}^{N} \beta_n^{1/2} exp(-\frac{\beta_n X_{f_n}^2}{2}) \quad (3)$$

Conditioning over the unknown precision and the known compressed samples, the full parameter distribution condition is obtained by combining the liklihood with Bayes' rule

$$p(\boldsymbol{X}_f|\boldsymbol{y}_t, \boldsymbol{\beta}, \beta_0) = \frac{p(\boldsymbol{y}_t|\boldsymbol{X}_f, \beta_0)p(\boldsymbol{X}_f|\boldsymbol{\beta})}{p(\boldsymbol{y}_t|\boldsymbol{\beta}, \beta_0)} \quad (4)$$

$$= (2\pi)^{-\frac{N}{2}}|\boldsymbol{\Sigma}|^{-\frac{1}{2}} exp\left\{-\frac{1}{2}(\boldsymbol{X}_f - \boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}_f - \boldsymbol{\mu})\right\} \quad (5)$$

which is a Gaussian distribution $N(\mu, \Sigma)$ where $\mu = \beta_0 \Sigma \boldsymbol{A}^T\boldsymbol{y}_t$ and $\boldsymbol{\Sigma} = (diag\{\beta_1, ..., \beta_N\} + \beta_0\boldsymbol{A}^T\boldsymbol{A})^{-1}$. μ is the point estimate of the signal while the diagonal elements of the covariance matrix provide "error bars" on the accuracy of the point estimate μ. From the above equations, all of the variables are known except $\boldsymbol{\beta}$ and β_0 which can be estimated using a marginal liklihood maximization technique such as the relevance vector machine [11]. For further depth, we refer the reader to [8].

IV. Multi-Resolution

The BCS framework requires a point-by-point modelling of the entire signal. In order to alleviate the complexity resulting from point-by-point modelling, we propose a multi-resolution wavelet hiearchical scheme in which we model a low-resolution wavelet-decomposed signal [12]-[15] rather than the original resolution signal as shown in Fig. 1. We then perform the BCS PU signal detection on the low-resolution

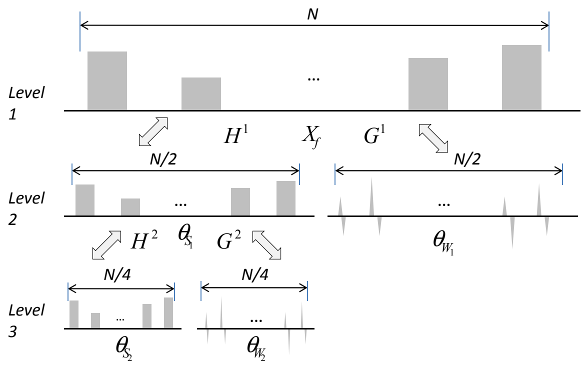

Fig. 1. Proposed Multi-Resolutional Bayesian Compressive Sensing (MR-BCS) wavelet hiearchical decomposition scheme

signal which can be done with significantly less computation. But this low-resolution processing sacrifices accuracy for computation efficiency, and so we introduce a coarse to fine refining process in which we use the Primary User signal location information obtained at the lowest resolution to refine only the signals of the Primary User, significantly improving accuracy. This two step process allows us to significantly reduce the computational complexity of BCS PU detection, while maintaining almost identical signal estimation accuracy.

A. Low Level Primary User Detection

Let's consider a block of our communication signal, $\boldsymbol{X}_f$, at the original signal level $L = 1$ with total length Q

$$X_k^1 = [X_f(k), X_f(k+1), ..., X_f(k+Q-1)]^T \tag{6}$$

where $Q = 2^{L_{max}-1}$, and L_{max} is the predefined lowest level of the wavelet hierarchy. The concatenation of these blocks make up our Nyquist Rate signal

$$\boldsymbol{X_f} = [(X_1^1)^T, (X_2^1)^T, ..., (X_{\frac{M}{Q}}^1)^T]^T \tag{7}$$

For each level of decomposition, the lower resolution signal (Θ_{S_L}, the energy scaling coefficients of the wavelet transform), can be obtained by lowpass filtering the current level signal with a half-band lowpass filter, and then downsampling the output by a factor of two. The highpass components (wavelet coefficiets, $\Theta_{W_i} \quad \forall i = 1, ..., L$) are the complements to Θ_{S_L} and can be obtained by using a highpass filter and then downsampling the output of the highpass filter by two. The wavelet decomposition can be done iteratively over a ladder filter bank structure as shown in Fig. 2 [14]. The decomposition from Level 1 to Level 2 is given by

$$\Theta_{S_1} = \mathbf{H}^1 X_k^1 \quad \text{and} \quad \Theta_{W_1} = \mathbf{G}^1 X_k^1 \tag{8}$$

where $\mathbf{H}^1$ and $\mathbf{G}^1$ are comprised of low-pass and high-pass filter responses respectively, mapping from level 1 to level 2 as shown in Fig. 1. The scaling and wavelet coefficients can be interpreted as the decomposition of the original signal onto

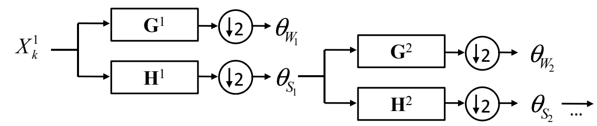

Fig. 2. Ladder filter bank implementation of the discrete wavelet transform

an orthonormal basis, so the reconstruction [14] from level 2 to level 1 is the sum of the orthogonal projections:

$$X_k^1 = (\mathbf{H}^1)^T \Theta_{S_1} + (\mathbf{G}^1)^T \Theta_{W_1} \tag{9}$$

The level of decomposition should be selected based on the anticipated lengths of the Primary User signals which must be detected. The underlying assumption is that the energy of a Primary User signal in X_f is well preserved in the scaling coefficients. Because each forward wavelet transform downsamples the signal by 2, the following constraint must be satisfied,

$$L \leq \log_2(B) + 1 \tag{10}$$

or else the scaling coefficients at level L will not be able to preserve the energy of the Primary User signal.

We implement our algorithm using a two-tap Haar wavelet transform, whose low-pass and high pass filters are as follows

$$\mathbf{H} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \quad \text{and} \quad \mathbf{G} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{11}$$

We choose to use the Haar wavelet transform because it is the simplest of wavelet transforms. This simplicity makes it easier to manipulate the basis to select only the wavelet coefficients which correspond to the Primary User signal, allowing us to perform the inverse wavelet transform on only those coefficients.

For a three-level transform, the block length is $Q = 4$ and the corresponding scaling and wavelet coefficients are obtained from the signal block X_k^1

$$\Theta^k = \begin{bmatrix} \Theta_{S_2}^k \\ \Theta_{W_2}^k \\ \Theta_{W_1}^k \end{bmatrix} = \begin{bmatrix} \mathbf{H}^2\mathbf{H}^1 \\ \mathbf{G}^2\mathbf{H}^1 \\ \mathbf{G}^1 \end{bmatrix} \begin{bmatrix} X_f(k) \\ X_f(k+1) \\ X_f(k+2) \\ X_f(k+3) \end{bmatrix} \tag{12}$$

and now we can express the relationship between the compressed measurements and the Nyquist rate signal as

$$\Theta^k = \Phi^T X_k^1 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} X_f(k) \\ X_f(k+1) \\ X_f(k+2) \\ X_f(k+3) \end{bmatrix} \tag{13}$$

As you can see from (13) in a three-level transform, every 4 points are fused into one low level scaling coefficient, Θ_{S_2}, which is 1x1 and two wavelet coefficients, Θ_{W_2} which is 1x1 and Θ_{W_1} which is 2x1. Expressing the entire length of the signal in terms of its scaling and wavelet coefficients, we get

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{\Phi}\boldsymbol{\Theta} + \boldsymbol{n}_f$$
$$= \boldsymbol{A}\begin{bmatrix}\Phi & 0 & \cdots & 0\\ 0 & \Phi & \vdots & \vdots\\ \vdots & \cdots & \ddots & 0\\ 0 & \cdots & 0 & \Phi\end{bmatrix}\begin{bmatrix}\Theta_{S_2}^1\\ \Theta_{W_2}^1\\ \Theta_{W_1}^1\\ \vdots\\ \Theta_{S_2}^{N/4}\\ \Theta_{W_2}^{N/4}\\ \Theta_{W_1}^{N/4}\end{bmatrix} + \boldsymbol{n}_f \quad (14)$$

where $\boldsymbol{y}_t$ is Mx1, $\boldsymbol{A}$ is MxN, $\boldsymbol{\Phi}$ consists of $\frac{N}{4}$ Φ's and is NxN, and $\boldsymbol{\Theta}$ is Nx1. Note that there is no dimension reduction in (14) because both the scaling and wavelet coefficients are being processed. In order to achieve the reduction in complexity and processing, we must set the wavelet coefficients, $\boldsymbol{\Theta}_{W_1}$ and $\boldsymbol{\Theta}_{W_2}$, to zero, under the assumption that the PU signal energy is preserved in the wavelet coefficients, which is true with the constraint of (10). This will allow us to perform detection only on the lowest level scaling coefficients $\boldsymbol{\Theta}_{S_2}$, which is $N/4$x1. To do so, we also need to modify our wavelet transform matrix, $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_S \; \boldsymbol{\Phi}_W]$, so that we only use the scaling function basis, $\boldsymbol{\Phi}_S$. We do this by selecting the 1^{st} column of each Φ, which corresponds to the scaling function basis of the wavelet transform matrix. So (14) becomes

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{\Phi}_S\boldsymbol{\Theta}_{S_2} + \boldsymbol{n}_f$$
$$= \boldsymbol{A}\begin{bmatrix}\Phi_S & 0 & \cdots & 0\\ 0 & \Phi_S & \vdots & \vdots\\ \vdots & \cdots & \ddots & 0\\ 0 & \cdots & 0 & \Phi_S\end{bmatrix}\begin{bmatrix}\Theta_{S_2}^1\\ \Theta_{S_2}^2\\ \Theta_{S_1}^3\\ \vdots\\ \Theta_{S_2}^{N/4}\end{bmatrix} + \boldsymbol{n}_f \quad (15)$$

where $\boldsymbol{y}_t$ is Mx1, $\boldsymbol{A}$ is MxN, $\boldsymbol{\Phi}_S$ is Nx$\frac{N}{4}$, and $\boldsymbol{\Theta}_{S_2}$ is $\frac{N}{4}$x1. We can now perform BCS on the lowest level scaling coefficients to detect the locations of the energy in the lowest resolution signal. By estimating the parameters of $\boldsymbol{\Theta}_{S_2}$ rather than the parameters of the original resolution Nyquist rate signal $\boldsymbol{X}_f$, we are able to significantly reduce the computation that BCS incurs in the initial point-by-point modelling of the entire signal. However, by reconstructing the low-resolution signal rather than the Nyquist rate signal, we lose out on the information contained within the wavelet coefficients $\boldsymbol{\Theta}_{W_1}$ and $\boldsymbol{\Theta}_{W_2}$, and so the parameters of the signal cannot be estimated as accurately. In order to improve the accuracy of this estimation while still capturing the computation saving benefits of low-level processing, we introduce a coarse to fine refining approach.

B. Coarse to Fine Mapping/Refining

In order to refine the reconstructed low-resolution signal, our objective is to map the wavelet coefficients of the detected Primary User signal to their corresponding low-resolution components. Because the low-resolution signal is a perfectly scaled version of the original signal, it preserves the location of the detected Primary User signals within the original resolution signal. This allows us to determine the indices of the wavelet coefficients which correspond to the detected Primary User signal, which we use in the next time step to reconstruct the portions of the signal occupied by Primary Users signal, and not the entire signal.

From (14) to (15), we set the wavelet coefficients, $\boldsymbol{\Theta}_{W_1}$ and $\boldsymbol{\Theta}_{W_2}$, to zero, and then proceded to perform BCS on the compressed samples to recover the $\boldsymbol{\Theta}_{S_2}$ in (15). Once the Primary User signal has been detected, we want to map the detected energy in the scaling coefficient to its corresponding wavelet coefficients only at that location. We're able to accurately identify the corresponding wavelet coefficients because of the preservation of proportions at the lowest resolution level. Assume that we have detected a Primary User signal at the i^{th} Φ_S location, we want to fully reconstruct the PU at that location only. To do so, we add the wavelet basis columns back into the corresponding scaling function basis:

$$\boldsymbol{\Phi}_S^R = \begin{bmatrix}0 & \cdots & 0 & \cdots & 0\\ \vdots & \ddots & \cdots & \cdots & 0\\ 0 & \cdots & \boldsymbol{\Phi}^{(4i+1):(4i+4)} & \cdots & 0\\ \vdots & \cdots & \cdots & \ddots & 0\\ 0 & \cdots & 0 & \cdots & 0\end{bmatrix} \quad (16)$$

where the superscript $(4i+1):(4i+4)$ denotes the selected row and column indices of the matrix. The indices which correspond to the detected signal at location i are located at multiples of 4 because of the 4 to 1 point mapping from the original resolution to the lowest resolution. The indices which do not correspond to any detected Primary User signal are zeroed out. Once the updated wavelet transformation matrix, $\boldsymbol{\Phi}_S^R$, has been obtained, it provides a direct relationship between the compressed measurements, $\boldsymbol{y}_t$, and the signals of the Primary Users.

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{\Phi}_S^R\begin{bmatrix}0\\ \vdots\\ \Theta_{S_2}^i\\ \Theta_{W_2}^i\\ \Theta_{W_1}^i\\ \vdots\\ 0\end{bmatrix} + \boldsymbol{n}_f \quad (17)$$
$$= \boldsymbol{A}^{(4i+1):(4i+4)}\boldsymbol{\Phi}^{(4i+1):(4i+4)}\begin{bmatrix}\Theta_{S_2}^i\\ \Theta_{W_2}^i\\ \Theta_{W_1}^i\end{bmatrix} + \boldsymbol{n}_f \quad (18)$$
$$(19)$$

where the superscript $(4i+1):(4i+4)$, denotes the row and column selected indices of $\boldsymbol{A}$ and Φ. Using the relationship between the Nyquist Rate signal and the wavelet/scaling coefficients, we obtain the final relationship between the compressed measurements and the Primary User signals

$$\boldsymbol{y}_t = \boldsymbol{A}^{(4i+1):(4i+4)}\boldsymbol{X}_f^{(4i+1):(4i+4)} + \boldsymbol{n}_f \quad (20)$$

We can then use this relationship, (20), to reconstruct only the Primary User signals, and not the entire signal. We have demonstrated here a three-level wavelet hierarchy as

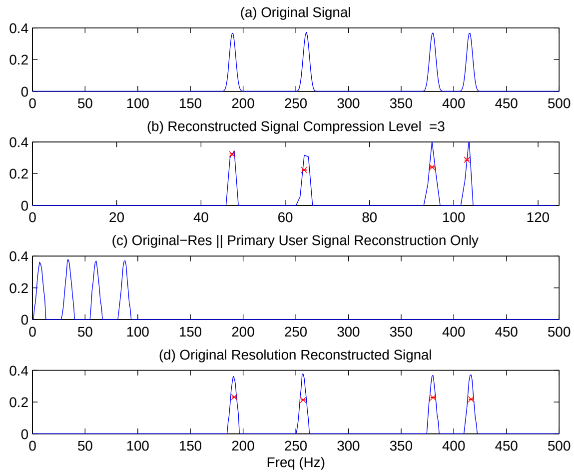

Fig. 3. MultiResolution Bayesian Compressive Sensing: K = 4, M = 140, N = 512, $\Gamma = 2 * 10^{-4}$. a) Original signal (b) Reconstructed lowest resolution signal (c) Original resolution reconstruction of only detected Primary User signal (d) Original resolution reconstructed signal

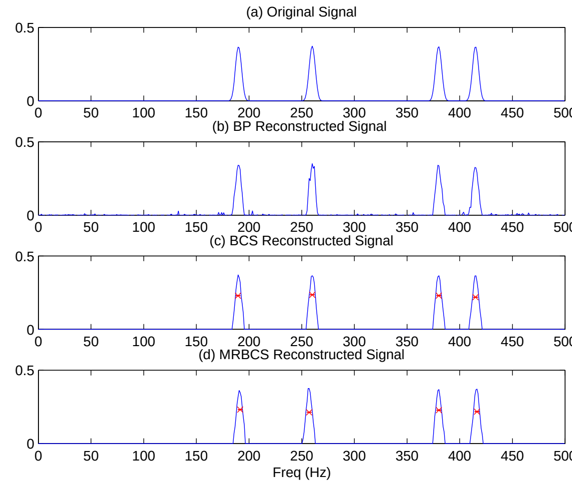

Fig. 4. Comparison of signal estimation using BP, BCS PU detection, and MRBCS PU detection for K = 6, M = 160, N = 512, $\Gamma = 2{*}10^{-4}$. (a) Original signal, (b) Reconstruction with BP, $t_{REC} = 1.3482$ secs (c) Reconstruction with BCS PU detection, $t_{REC} = 0.3102$ secs (d) Reconstruction with MRBCS PU detection, $t_{REC} = 0.0551$ secs

an example but a wavelet hierarchy for more levels can be derived in a similar manner, as long as the constraint in (10) is satisfied.

V. Simulations

In this section, we present some simulation results to demonstrate the advantages of our MRBCS algorithm. We consider a radio scene occupied by $j \in [0, K]$ users, each occupying arbitrary disjoint bands of equal bandwidth $B = 10Hz$. Each user employs Binary Phase Shift Keying (BPSK) and the carrier frequency is randomly generated between 0-500 Hz. We model measurement noise as a zero-mean Gaussian with $\sigma = 0.005$. In each of our simulations, the total occupied bandwidth is $2jB << f_{nyq}$ and so the received signal is sparse in the frequency domain. We use a 512 point unitary DFT matrix mapping the time domain signal to the frequency domain where $\mathbf{X}_f$ is sparse, so the Nyquist rate signal has length N=512. We demonstrated the algorithm on a 512 Hz signal for simplicity but the same principles apply for a wideband signal.

We compare the performance of the MRBCS algorithm described in this paper against our BCS algorithm in [8] and Basis Pursuit (BP) [5], an inversion method which uses linear programming. The three metrics we will use to compare them are 1) Computational Time, which measures the computational complexity of the algorithm, 2) Reconstruction Error, which measures the accuracy of the inversion 3) Minimum number of samples to satisfy Γ, which measures how much compression can be achieved.

We begin by demonstrating how our Multi-Resolution approach to signal estimation further reduces the computational complexity of our BCS PU detection, while maintaining comparable performance in Fig.3. First, we process a lower resolution signal to obtain an initial estimate of where the Primary User signals are. Fig. 3b shows the reconstructed lowest resolution scaling coefficients $\mathbf{\Theta}_{S_2}$, where adequate reconstruction is achieved with 98 samples and takes .008813 seconds to perform. Notice how the location of the Primary User signal is scaled by a factor of 4, but otherwise perfectly preserved. The lowest resolution scaling coefficients do not accurately represent the signal because of the loss of the wavelet components, $\mathbf{\Theta}_{W_1}$ and $\mathbf{\Theta}_{W_2}$, which were set to 0 in order to reduce the complexity. However, once the locations of the Primary User Signals have been detected at the lowest resolution, we can map the signal to its corresponding wavelet coefficients, and zero out the rest of the scaling coefficients where no Primary Users have been detected. This step is shown in Fig. 3c, where only the detected Primary User signals are reconstructed (We also reconstructed a small buffer area around the detected signal so that the reconstructed Primary User signals did not overlap). The hyperparameters of the original resolution Primary User signals are then optimized and the threshold Γ is met with 105 samples and requires .046320 seconds to perform. Once the Primary User signals have been fully reconstructed, we map the reconstructed Primary User Signals, detected at the lowest resolution as shown in Fig. 3b, to their scaled locations at the original resolution as shown in Fig. 3d.

Next we make the comparison between MRBCS, BCS PU detection [8], and the traditional BP reconstruction [5] shown in Fig. 4. As you can see from the figure, BCS PU detection is much cleaner than BP, as the relative reconstruction error of BCS is less than a third of the error incurred by BP as shown in Table I. In addition, because BCS PU detection refines only the Primary User signals, and not the entire communications signal as BP does, it achieves this improved result in less than 20% of the time that it takes BP to reconstruct the entire signal. Finally, BCS PU detection also yields a confidence metric in the form of "error bars", this allow it to decide in the next instance whether to increase or decrease the number of samples to meet certain application requirements. This is a feature that BP does not have.

As discussed in Sect. IV, there is still room to reduce

TABLE I
COMPARISON OF INVERSION TECHNIQUES

Inversion Technique	**Computation Time** (s)	**Reconstruction Error** 200 samples: $\frac{\|\|\hat{X}_f - X_f\|\|}{\|\|X_f\|\|}$	**Sample $_{min}$ satisfying Γ**
Basis Pursuit (BP)	1.3482	.2381	223
Bayesian Compressive Sensing (BCS)	.3102	.0891	164
Multi-Resolution BCS	.0551	.0975	105

the computational complexity and increase the compression rate of the BCS approach using a Multi-Resolution approach. Comparing the two reconstructions visually in Fig. 4(c) and Fig. 4(d), we can see that the reconstruction accuracy is virtually the same, and the actual reconstruction errors in Table I confirm this visual hypothesis. MRBCS incurs a reconstruction error which is less than 10% greater than that incurred by BCS. This error is mostly due to the location mapping; because 4 points are being mapped to 1, and the detected location at the lowest resolution is considered at the center of the 4 points at level 1 resolution, the maximum offset would be 2 points. However, with this marginal increase in reconstruction error, MRBCS is able to significantly increase computational speed, performing the same reconstruction in less than 18% of the time that BCS requires. Also, because MRBCS is not performing point-by-point modelling on the entire signal, it is able to achieve a higher compression rate, requiring only 10.3% of the Nyquist Rate samples to achieve Γ, much less than the 16% of Nyquist Rate samples that BCS requires. These results demonstrate that our original motivation of reducing computational complexity of Compressive Sensing in Cognitive Radios is accomplished with a MRBCS framework.

VI. CONCLUSIONS

In this paper, we highlighted the potential for CS in CRs to alleviate the sampling limitations of modern analog-to-digital hardware. We then discussed how CS is a powerful technique, but the resulting increase in computation/complexity is non-trivial, especially in a power-constrained mobile Cognitive Radio. This motivated our development of the MRBCS scheme.

One of the key observations of this paper was the fact that when applying CS to CRs, the fundamental task of the CR remains the same: detect the presence of Primary Users. At this point, our work diverges from the prior applications of CS to Primary User Detection and establishes a new precedent. Because the goal of the CR is not to reconstruct the signal, rather it is to estimate the presence of Primary Users, the reconstruction stage can be completely eliminated. In order to accomplish this, we used a Bayesian formulation to estimate the parameters of the underlying signal from compressed measurements. We demonstrated in multiple simulations how the error bars provided by such a Bayesian formulation could be used to design an iterative optimization strategy to acquire the minimum number of samples in a dynamic sparsity model without any *a priori* assumptions.

While powerful, the BCS framework requires a point-by-point modelling of the entire signal. To further alleviate this complexity and allow for even greater signal compression, we used a multi-resolution wavelet hiearchical scheme in which we model a low-resolution wavelet-decomposed signal rather than the original resolution signal, which we called Multi-Resolution Bayesian Compressive Sensing (MRBCS). We demonstrated in simulations how this strategy required significantly less computation, performing the same signal estimation in less than 20% of the time it took the BCS framework to accomplish the same task. In addition, because MRBCS is not point-by-point modelling the entire signal, it is able to accomplish the same reconstruction with fewer samples, allowing it to achieve a higher compression rate and achieve our primary goal: significantly reducing the computational complexity associated with Compressive Sensing in Cognitive Radios.

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